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MANAGEMENT STRATEGIES IN FIXED-STRUCTURE MODELS OF COMPLEX ORGANIZATIONS

Crayton C. Walker and Alan E. Gelfand

1. Introduction

We claim that the study of binary switching nets is useful in enhancing our understanding of the dynamics of complex real world organizations. To date examination of such nets has been largely in biological contexts. Since our purpose in this paper is an examination of some factors that contribute to useful network behavior we sharply limit our discussion of the literature. Some of the literature on ensembles of switches along with tangentially relevant studies such as those on formal nerve nets and epidemiological models are excluded. We first describe the nets considered here, their behavior, and, briefly, some interpretations of them as models before moving to the substance of the paper.

The objects of interest are networks of a finite number N of logic elements. Each element has k binary inputs and computes a specific, determinate binary function of those inputs at net time t. Each input is

connected to some element's output, according to the particular net's fixed, unchanging structure, which is typically unknown. At net time t+1 the former output values become the input values on the appropriate elements, and the process repeats under these possibly new conditions. If by the state of the net at time t we understand the ordered set of N element outputs at time t, the action of the net is to move in a determinate fashion from one net state to another as time proceeds. There are clearly 2^N distinct net states. Therefore from some arbitrary initial net state the net eventually must encounter a state it had shown before. Doing so, it must thereafter repeat the intermediate sequence of states. Such a sequence of states is called a cycle. If the number of distinct net states in a cycle is called the cycle length, it is apparent that cycle lengths may range from one to 2^N.

The biological analogy devised by Kauffman [2] interprets the net as a cell and each element as a gene, which produces, or does not produce, a protein. The net structure is the fixed but unknown web of influence that the various genes have on one another by virtue of their products. The cycle in which a given net is moving is therefore interpreted as the specific tissue type being computed by that genetic net. The cell is thus the product of a collection of interlaced switches operating over time.

The organizational analogy appropriate for our purposes interprets the switching net as a control system imbedded in an organization. A given model might encompass virtually a whole organization, or only some part of it; there may be several control systems in any given organization. The net elements correspond to points in the organization where control information is used. The use can be executive, evaluative, productive, and so forth, depending on the particular activity modeled. Thus the elements could variously be seen as governmental bureaus, machines, or individuals. For example, the net elements could collectively represent a group of middle managers carrying out organizational routine, staff personnel reaching a decision, foremen and their crews producing goods, or mixtures of these functional types. The network structure represents the control relations which exist among the elements, that is, the channels through which actual control information is passed in the system. The responses of a control point to the possible contingencies presented to it by its sources of information will be described by a function or mapping. We will later interpret equivalence classes of these mappings as types of management strategy. If the elements are seen as machines our net is essentially a production process, which has N possibly different constituents each of which is either being produced or not. The cycle models the product,

the states which precede the cycle model, the start-up period, and the entire set of cycles gives the inventory of products the particular control structure and particular set of constituents is capable of producing.

In our model the network structure is taken to be fixed but unknown in detail. This presumption is appropriate for our objective of modeling complex organizations. Certainly one important source of complexity in real organizations is their intricate structure. Even though an organizational control grid might be relatively fixed and in principle specifiable, the time required to know its details and thereby to make a prediction of system response to some proposed change, might often exceed the time within which that prediction were needed. Therefore it seems to us that the practice of management requires a body of knowledge concerning what can be predicted under conditions of structural uncertainty. One method of accommodating that uncertainty is to study appropriate ensembles of structure. This method refocuses analytic interest on the behavioral properties of the set of control structures so generated. The data we cite and the applications we suggest refer therefore to ensembles of structures, in particular to ensembles generated by all possible assignments of inputs to affecting elements.

It is clear that control systems in larger domains such as industries or economic sectors may well be

usefully examined through network models. However, our interpretation will focus on what can be accomplished within single organizations by varying strategies, or by maintaining different intensities of use of a strategy.

A fair question to ask is whether the behavior of the simple nets is such as to make them at all possible as models of real world organizations. If N=1000 then cycle lengths of a net may be on the order of $2^{1000} \simeq 10^{300}$. Using any plausible assumptions as to how fast the net moves from state to state, such a net could model no repetitive real world phenomenon. If ensembles of simple nets do show themselves as candidate models, then at the very least we can expect some suggestions as to what factors may not be crucial in understanding and controlling large systems.

The findings are that under certain broad conditions, simple nets typically show transient and cycle lengths that are empirically reasonable. In fact they demonstrate additional desirable behavioral properties which are discussed at greater length at the end of Section 2. Two suggestions for organizational theory follow. (1) If it is reasonable to assume that elements with simple properties are likely to be easier to construct or otherwise be more readily available than are more complicated elements then systems using the simpler elements, other things equal, would have a survival advantage over

systems using more complex elements. Since, as we have noted, some useful large scale characteristics are the typical result of certain very simple small scale properties, we might expect to find such small scale properties in real many-element systems which have been decisively shaped by competitive survival regimes. That is, we might expect organizations subject to evolutionary influences to make use of those arrangements which can, in a sense, exploit structural indifference. (2) As a corollary, for those involved in organizational design or intervention, the suggestion is that details of structure in the large, in appropriate contexts, may not be a crucial parameter.

As to what those appropriate contexts are, Walker and Ashby [6] found that increasing the sameness of the entries, i.e., the internal homogeneity, of the elements' function table tends to decrease the length of cycles. Kauffman found the number of inputs to be importantly related to both cycle length [2] and to their stability [3]. In his latter paper he describes a powerful explanatory property called forcibility which is also implicated in cyclic behavior. Babcock [1] then showed that the two measures, internal homogeneity and forcibility, are related. In Section 2 we correct and extend the relationship and examine implications of the relationship for many-element switching net models. In a later paper we generalize yet another measure, the

notion of a threshold for a mapping and consider its interface with these two measures along with implications for a switching net. In the final section we interpret forcibility and internal homogeneity with respect to managerial strategy.

2.1. Definitions and Notation

As described in our introduction, we wish to examine elements in a switching net whose behavior is guided by Boolean transformations on their inputs. If we assume that a particular element has k inputs then the transformation (mapping) prescribes for each vector (a vector of 0's and 1's) the next state (0 or 1) for the element. For convenience a transformation is usually presented in a table arranged in lexicographic order as illustrated in Table 1 below.

x_3	x_2	x_1	m
1	1	1	e_1
0	1	1	e_2
1	0	1	e_3
0	0	1	e_4
1	1	0	e_5
0	1	0	e_6
1	0	0	e_7
0	0	0	e_8

Table 1: General Representation of a Transformation on Three Inputs

Thus a mapping on k inputs requires specification of its value for each of its 2^k possible input vectors and moreover there are a total of 2^{2^k} possible mappings. It may be convenient at times to denote a mapping m as a function of its inputs, i.e., $m(x_1,x_2,\ldots,x_k)$.

By definition, a mapping is forcible on a given input when a given state of the input "forces" the output of the mapping to a single value regardless of the values of the other inputs. This given state is called the forcing state. If an input is forcing on both states then the mapping is either constant (trivial) or has half "1"'s and half "0"'s. In the former case all inputs are forcing on both states while in the latter case the mapping must be forcing only on the one input. Since forcibility with only one input is trivial we restrict attention to the case where the number of inputs $k \geq 2$. The forced value of an element is that value to which it is forcible.

If an element is forcible on more than one input line, its forced value is identical for all the inputs on which it is forcible. This is apparent since forcibility on a particular input implies that the mapping assumes a forced value on at least 2^{k-1} of the 2^k input vectors. It is easy to verify that of the 16 mappings on two input coordinates ten are forcible on both coordinates (including the two trivial constant

mappings), four are forcible on one and two on neither.

Notice that forcibility as we formulate it here is a "local" property; we examine mappings for one element in a system. In Section 2.4 we consider ramifications of forcibility for the entire net. Internal homogeneity, denoted henceforth by I is defined as the larger of the number of entries of "0" and of "1" in the table of values of a mapping, i.e., I = max(#0's, #1's) and hence $2^{k-1} \leq I \leq 2^k$. We denote by $N_k(i)$ the number of mappings with I = i.

2.2. Enumerating the Forcible Mappings

The major result of this section is Theorem 1. In order to develop this result we begin with several elementary lemmas.

Lemma 1: $N_k(i) = 2\binom{2^k}{i}$ for $2^{k-1} < i \leq 2^k$ and if $I = 2^{k-1}$, $N_k(2^{k-1}) = \binom{2^k}{2^{k-1}}$. Note: Henceforth, the symbol $\binom{n}{r}$, n and r integers, denotes the number of combinations of n distinct objects taken k at a time, i.e., $\binom{n}{r} = \frac{n!}{r!(n-r)!}$.

Pf.: Obvious.

Corollary 1: The mean internal homogeneity for mappings on k inputs is

$$2^{k-1} + 2^{k+1-2^k}\binom{2^k-1}{2^{k-1}-1}.$$

Proof: We seek

$$2^{-2^k}\sum_{i=2^{k-1}}^{2^k} i\, N_k(i) = 2^{-2^k}\left[\sum_{i=2^{k-1}+1}^{2^k} i\; 2\binom{2^k}{i} + 2^{k-1}\binom{2^k}{2^{k-1}}\right]$$

$$= 2^{-2^k}\left[2^k\sum_{i=2^{k-1}+1}^{2^k} 2\binom{2^k-1}{i-1} + 2^k\binom{2^k-1}{2^{k-1}-1}\right]$$

$$= 2^{-2^k}\left[2^k\sum_{j=2^{k-1}}^{2^k-1} 2\binom{2^k-1}{j} + 2^k\binom{2^k-1}{2^{k-1}-1}\right]$$

$$= 2^{-2^k}\left[2^k\sum_{j=0}^{2^k-1}\binom{2^k-1}{j} + 2^{k+1}\binom{2^k-1}{2^{k-1}-1}\right]$$

$$= 2^{-2^k}\left[2^k\cdot 2^{2^k-1} + 2^{k+1}\binom{2^k-1}{2^{k-1}-1}\right]$$

$$= 2^{k-1} + 2^{k+1-2^k}\binom{2^k-1}{2^{k-1}-1}. \quad \square$$

We wish to enumerate $\Gamma(k,i,j)$ which is defined to be the number of mappings on k inputs, with internal homogeneity i, which are forcing on exactly j of the k

inputs. We will develop a recursive procedure which calculates $\Gamma(k,i,j)$ for $j>0$. We then can obtain $\Gamma(k,i,0)$ by

$$\Gamma(k,i,0) = N_k(i) - \sum_{j=1}^{k} (k,i,j).$$

From $\Gamma(k,i,j)$ we can calculate the number of mappings on k inputs with internal homogeneity i, which are forcing on at least (at most, etc.,...) j inputs by simple summation. Moreover we can calculate the number of mappings on k inputs forcing on exactly j inputs, $\Gamma(k,j)$ again by summation, i.e.,

$$\Gamma(k,j) = \sum_{i=2^{k-1}}^{2^k} \Gamma(k,i,j).$$

If we seek the density of any of these collections of mappings we standardize by either $N_k(i)$ or 2^{2^k} depending on the base of reference.

First we pose the following questions. Given k and j how large must i be in order to allow the possibility of j forcing inputs? Reciprocally, given k and i, how many forcing inputs are possible, i.e., how large may j be? The answer is expressed in the following lemma.

Lemma 2: (i) Given k and j, we can't have j forcing inputs unless $i \geq 2^k - 2^{k-j}$.

(ii) Given k and i, a mapping may have at most $[k - \log_2(2^k - i)]$ forcing inputs where $[x]$ denotes the greatest integer in x.

Pf.: Result (ii) follows from result (i) by solving the inequality for j so that we need only establish (i). To see (i) we note that in order that a mapping be forcing on one input we need at least half of the entries to assume the forced value. From the lexicographic order, if the mapping is to be forcing on a second input we need at least half of the remaining entries to assume the forced value. Building inductively we need $i \geq 2^{k-1} + 2^{k-2} + \ldots + 2^{k-j}$ in order that we have a sufficient number of entries at the forced value to allow j forcing inputs. Simplifying this inequality produces the result in (i). ▯

For convenience, then, we set $\Gamma(k,i,j) = 0$ if $i < 2^k - 2^{k-j}$.

We are now ready to calculate $\Gamma(k,i,j)$. In the following lemma we take care of the two extreme cases.

Lemma 3: (i) $\Gamma(k,2^k,j) = 2,\ j=k$

$= 0,\ j \neq k$

(ii) $\Gamma(k,2^{k-1},j) = 2k,\ j=1$

$$= \binom{2^k}{2^{k-1}} - 2k,\ j=0$$

$= 0$, otherwise.

Pf.: (i) Obvious.

(ii) Again considering the lexicographic order there will be exactly two mappings forcing on the first input, i.e., the upper half 1's and the lower half 0's or vice versa. But the selection of the first input is but one of k choices so in total $\Gamma(k,2^{k-1},1) = \binom{k}{1}\cdot 2 = 2k$. It is impossible by Lemma 2 to have more than one forcing input (this is clear directly!) and thus

$$\Gamma(k,2^{k-1},0) = N_k(2^{k-1}) - 2k = \binom{2^k}{2^{k-1}} - 2k \text{ by Lemma 1.} \quad \square$$

We now proceed to the major result.

Theorem 1: If $2^{k-1} < i < 2^k$ and $j > 0$

$\Gamma(k,i,j) = \binom{k}{j}2^{j+1}\gamma(k,i,j)$ where

$$\gamma(k,i,j) = \begin{cases} \binom{2^{k-j}}{2^k-i} & \text{if} \quad i<2^k-2^{k-j}+2^{k-j-1} \\[2ex] \dfrac{\Gamma(k-j,i-2^k+2^{k-j},0)}{2} & \text{if} \quad i>2^k-2^{k-j}+2^{k-j-1} \\[2ex] \binom{2^{k-j}}{2^{k-j-1}}-2(k-j) & \text{if} \quad i=2^k-2^{k-j}+2^{k-j-1} \end{cases}$$

(Note: Conventionally we take $\Gamma(1,1,0) = 0$, $\Gamma(1,1,1) = 2$ for consistency of notation since we have restricted $k \geq 2$. Nonetheless these conventions coincide with our definitions in this trivial case.)

Pf.: In our proof we consider the first j inputs. Hence the $\binom{k}{j}$ term is needed to adjust for the arbitrary selection of j inputs. The 2^{j+1} term arises from there being 2 choices for the forced value along with 2^j possibilities for the j forcing inputs in selecting which value for each of the inputs forces to the forced value.

The γ term provides the most crucial counting. As in the argument for Lemma 2 we will need $2^k - 2^{k-j}$ entries at the forced value in order that the first j inputs will be forcing. The remaining entries at the forced value, must be arranged to force <u>none</u> of the remaining inputs. Hence the problem is collapsed to examining mappings on a reduced number of inputs k-j with $i-2^k+2^{k-j}$ entries at the forced value and 2^{k-i} at the nonforced value. If $i-2^k+2^{k-j} < \frac{1}{2}(2^{k-j}) = 2^{k-j-1}$ then any arrangement of these entries will fail to force another input, i.e.,

$\gamma(k,i,j) = \binom{2^{k-j}}{2^k-i}$. If $i-2^k+2^{k-j} > \frac{1}{2}(2^{k-j}) = 2^{k-j-1}$ then

to insure that these entries will not force another input we need $\gamma(k,i,j) = \Gamma(k-j, i-2^k+2^{k-j}, 0)/2$. (The reason for the divisor of 2 is that we do not at this point have two choices for the forced value which we had in the original counting). Finally if $i-2^k+2^{k-j} = \frac{1}{2}(2^{k-j}) = 2^{k-j-1}$ we are precisely in the situation of Lemma 3, and need

$\Gamma(k-j, 2^{k-j-1}, 0) = \binom{2^{k-j}}{2^{k-j-1}} - 2(k-j)$. Rearranging the inequalities on i, k and j produces the exact form of the theorem. $\square$

Note that this construction enables us to build up $\Gamma(k,i,j)$ recursively. The following tabulations given in Table 2 may be verified both directly and from Theorem 1. The row totals provide $\Gamma(k,j)$ while the column totals provide $N_k(i)$.

Corollary 2: (i) $\Gamma(k, 2^k-1, k) = 2^{k+1} = N_k(2^k-1)$

(ii) $\Gamma(k, 2^k-2, k-1) = k \cdot 2^k = \Gamma(k, k-1)$.

Pf.: (i) and the left equality in (ii) are obvious by calculation. The right equality in (ii) follows since Lemma 2 requires $i \geq 2^k-2$ in order to allow k-1 forcing inputs and (i) shows $\Gamma(k, 2^k-1, k-1)=0$.

(i) of Corollary 2 implies that any mapping on k inputs with internal homogeneity equal to 2^k-1 will be forcing on all inputs; (ii) implies that the only mappings on k inputs which will be forcing on k-1 of them have internal homogeneity equal to 2^k-2. A result of this latter type has been discussed by Newman and Rice [4].

2.3. Some Convenient Approximations

For analytical purposes the results of Theorem 1 are a bit awkward to use. Although constructively we

k=2

j \ i	2	3	4	
0	2	0	0	2
1	4	0	0	4
2	0	8	2	10
	6	8	2	16

k=3

j \ i	4	5	6	7	8	
0	64	64	8	0	0	136
1	6	48	24	0	0	78
2	0	0	24	0	0	24
3	0	0	0	16	2	18
	70	112	56	16	2	256

k=4

j \ i	8	9	10	11	12	13	14	15	16	
0	12,862	22,752	15,568	7,840	2,568	640	16	0	0	62,246
1	8	128	448	896	1,024	288	64	0	0	2,856
2	0	0	0	0	48	192	96	0	0	336
3	0	0	0	0	0	0	64	0	0	64
4	0	0	0	0	0	0	0	32	2	34
	12,870	22,880	16,016	8,736	3,640	1,120	240	32	2	65,536

Table 2: $\Gamma(k,i,j)$ for $k=2,3,4$.

have developed a recursion to calculate $\Gamma(k,i,j)$, since $\Gamma(k,i,0)$ is calculated by subtraction and yet is also needed for subsequent $\gamma(k,i,j)$ it becomes very difficult to employ this theorem to bound $\Gamma(k,i,j)$ and hence $\Gamma(k,j)$.

We would like a simple bound on the number of mappings forcing on at least j inputs. In our notation the quantity we are bounding is

$$\sum_{j'=j}^{k} \Gamma(k,j').$$

A convenient result is given in Theorem 2.

Theorem 2. $\displaystyle\sum_{j'=j}^{k} \Gamma(k,j') \leq 2^{j+1}\binom{k}{j}2^{2^{k-j}}$.

Pf.: The 2^{j+1} and $\binom{k}{j}$ terms arise for the same reasons as in the proof of Theorem 1. We may therefore focus on the first j inputs and assume that "1" is the forcing input value and that "1" is the forced value as well.

If the first $2^{k-1} + 2^{k-2} + \ldots + 2^{k-j} = 2^k - 2^{k-j}$ of the 2^k entries in the mapping are at the forced value "1" then regardless of the remaining 2^{k-j} entries, we will have at least j forcing inputs. (Recall the proof of Lemma 2). These remaining entries may be selected in $2^{2^{k-j}}$ ways completing the result. $\square$

Considerable double counting is involved in obtaining this upper bound. Each mapping forcing on say $j' > j$ inputs will be counted once for each of the

$\binom{j'}{j}$ subsets of j inputs.

When j=1 we have a bound on the total number of mappings forcible on one or more input line. Kauffman [3] has offered such a bound in his Theorem 2, p. 105. Unfortunately his expression is incorrect as given. The sums must be replaced by products. His subsequent Theorems 3, 4 and 5 suffer the same error. In any event our bound which is

$$4k \cdot 2^{2^{k-1}}$$

is easily seen to be tighter than his corrected bound.

Let us employ this bound to prove a result motivated by Table 2. Table 2 suggests that the density of mappings not forcing on any inputs tends to one as k increases. Equivalently we conjecture that the density of forcible maps goes to zero as k grows large. The conjecture is trivially proved since

$$\frac{\sum_{j=1}^{k} \Gamma(k,j')}{2^{2^k}} \leq \frac{4k \cdot 2^{2^{k-1}}}{2^{2^k}} = \frac{4k}{2^{2^{k-1}}} \to 0$$

as $k \to \infty$.

We restate this result as a corollary.

Corollary 3: The density of forcible maps (i.e., maps forcing on at least one input line) tends to zero as the number of inputs increases.

In concluding this section we obtain a companion result to Corollary 2 by bounding the mean number of forcing inputs for mappings on k inputs.

Corollary 4: The mean number of forcing inputs for mappings on k inputs is at most

$$2^{-2^{k-1}} \cdot 4k^2 .$$

Pf.: We seek

$$2^{-2^k} \sum_{j=0}^{k} j\Gamma(k,j)$$

$$\leq 2^{-2^k} k \sum_{j=1}^{k} \Gamma(k,j)$$

$$\leq 2^{-2^k} k \cdot 4k \cdot 2^{2^{k-1}} = 2^{-2^{k-1}} \cdot 4k^2 . \quad \square$$

2.4. Network Behavior with Regard to Internal Homogeneity, Forcibility and Number of Inputs

If maps are classed by k, sharp differences are found to exist, particularly for forcibility. From Table 2 it can be seen that the density of forcible maps drops sharply as k is increased, from 0.875 at k=2 (and from 1.0 at k=1) to 0.05 at k=4. From Theorem 2 it is known that the limiting density is 0. The mean fraction of forcing inputs per map declines at approximately the same rate: from all inputs forcing at k=1 to about 1.5% of inputs forcing at k=4. In fact from Corollary 4 we see that the mean fraction of forcing inputs per map on

k inputs is at most $k^{-1} \cdot 2^{-2^{k-1}} \cdot 4k^2$ which clearly approaches zero very quickly as k increases. The drop is sharp enough to justify a simple summary: classified by number of inputs, only Boolean maps with fewer than four inputs are forcible. That is, if mappings are selected equiprobably, nets using elements with four or more inputs will have very low densities of forcing elements.

This state of affairs parallele the behavioral distinctions observed by Kauffman. In [2] and [3] he studies functionally heterogeneous nets, selecting mappings at random for each element. Classifying nets by number of inputs, k, he concludes that these nets are behaviorally implausible or plausible as models of real world phenomena according to whether $k \geq 4$ or $k \leq 3$ respectively. More specifically he makes the following observations on k=2,3 and N nets. For k=2 nets,

(i) As we have observed, the density of elements forcible on one or more input lines is high.

(ii) Cycle and transient lengths are minimized. State cycle lengths are typically $\sqrt{N}$, N the number of elements. A net with 10,000 elements and $2^{10,000}$ states typically cycles through a miniscule 100 states. The number of cycles is also typically $\sqrt{N}$.

(iii) The behavior tends to be highly stable. If a net is perturbed by changing the value of an arbitrary element as it passes through a state cycle, for 90

to 95% of possible perturbations the net returns to the cycle perturbed.

(iv) The behavior is highly localized. This is observed in terms of marked restricted local reachability. Under perturbation a cycle will typically flow to only about 5 or 6 other cycles.

(v) Only two of the 16 possible mappings are not forcible on either input. As established by data available in Walker [7], nets built using exclusively these mappings have enormously long state cycles.

For k=3 nets, cycle lengths do become a bit longer but still tend to be well below the 2^N limit. Moreover in the restricted set of mappings on 3 inputs which are forcing on at least one (which includes nearly half the mappings) such nets behave as in the k=2 case. For k=N, a totally connected net, the density of forcible functions is very low and cycle lengths are quite long. The expected cycle length is $\sqrt{2^N}$. Perhaps surprisingly the expected number of distinct cycles is only N/e. Totally connected nets show little or no homeostatic tendency to return to the cycle from which they were perturbed and also show no restricted local reachability.

Internal homogeneity is not sharply affected by the number of inputs. Again grouping by k, the mean internal homogeneity per map expressed as a fraction, i.e., $i/2^k$, can be found from Table 2 to decline from about 69% at k=2 (or 75% at k=1) to about 60% at k=4. From Corollary 1

the mean fractional internal homogeneity is (dividing by 2^k)

$$\frac{1}{2} + 2^{1-2^k}\binom{2^k-1}{2^{k-1}-1}$$

which is approximately

$$\frac{1}{2} + (2\pi 2^k)^{-\frac{1}{2}}$$

as k grows large. Thus the limiting mean fractional internal homogeneity is 1/2. Walker and Ashby [6] studied functionally homogeneous nets (where all elements have the same output function). They found cycle lengths to vary widely with the mapping used, and that if nets were grouped by the internal homogeneity, cycle and transient lengths tended to decrease with increasing internal homogeneity. Increasing internal homogeneity is also related to increased production of the shortest cycles. These conclusions are intuitively reasonable, since it is clear that the more likely elements are to send the same value to these elements they affect, the more likely it is that affected elements will remain fixed at a given output value. Each element that is fixed in value reduces the number of net states that can be explored by half. The salient point here is that the reduction in cycle and transient lengths downwards is severe. The observed lengths are orders of magnitude below the 2^N limit.

Finally Table 2 also shows that for a given number of inputs internal homogeneity does affect forcibility. It is uniformly the case that increasing I increases the density of maps with that value of I that are forcible. While internal homogeneity and forcibility are certainly not identical concepts, they are highly correlated. Increasing the number of inputs increases the mutual dependence.

3.1. Organizational Implications and Interpretations

We now return to the organizational analogy developed in the introductory section. Recalling that discussion, one crucial characteristic of the net models we are using should be noted. All elements' inputs are connected to element outputs. Net operation, therefore, is not affected by external information. Thus the organizational systems modeled must be assumed (i) effectively insulated from the environment as perhaps by inventory policy in the case of a production system, or by high echelon planning as in the case of a middle level management system, (ii) tied to the environment by a relatively slow or infrequently acting control loop, as might be the case for stockholder influence in a corporate model, or (iii) such that the environment itself is so intimately linked to the organization that it can be appropriately included in the net.

In understanding the dynamics of large real world organizations one suggestion of the preceding section is the following. For organizations that have been shaped by strong, competitive evolutionary pressures it would be expected that elements with three or fewer control inputs should be found in relative abundance. This assumes, as we have mentioned, that simpler functions are more readily constructed and maintained. Note that we do not claim that evolutionary regimes must produce organizations with low k values, but only that organizations so constructed have some survival advantages, on the average, over those not so constructed. For those tasks requiring elements with high k values, the suggestion is that here survival advantage would accrue to organizations built (i) with high densities of very forcible elements (and thus showing high internal homogeneity) or (ii) with fortunate structural arrangements which are easily maintained once in place.

Application of these findings in management presupposes that the manager or consultant has determined that with respect to net structure and assignment of element function, specific design is inappropriate. Perhaps the organization is too large to allow specific design within the time available, or a structural reorganization is thought to be excessively expensive or hazardous. If such is the case design considerations are reduced to

two: in our terms, the number of inputs, and the mapping subclasses from which element mappings are allowed to be chosen. If the number of inputs can be varied it is an attractive control option. As we have seen, k, by itself, can strongly shape system behavior into modes in which useful outcomes are at least possible. Should the number of inputs not be accessible, the designer is limited, in our scheme, to modifying the mapping subclass. That is, he or she then has the task of specifying parameter values which appropriately bias functional densities. Following our discussion above, and assuming that the designer determines that he is dealing with a high k (≥ 4) net, he would operate to increase the density of forcible maps in the net, either directly, or through internal homogeneity, whichever is the better choice in the particular circumstances.

3.2. Interpretations of Internal Homogeneity and Forcibility as Managerial Strategy

Some interesting interpretations of internal homogeneity and forcibility with regard to management policy are possible. Consider a manager attempting to increase internal homogeneity. He might address the people occupying the net elements as follows: "Look carefully at your sources of control information (k inputs), and at each possible control order they can confront you with (the 2^k lines in the mapping). I want

you to determine the very fewest of those orders on which action is absolutely necessary, and to take no action otherwise." This control style is, of course, management by exception. Consider the approach when he intends to increase forcibility: "I want you to look carefully at your mission and potential sources of control information. Arrange your activities by determining the largest group of sources whose orders individually are warrant enough to take action." This control style is harder to characterize, but it amounts to what might be called management by sufficient indication, or management by priority.

To summarize using these interpretations, the evidence available allows us to understand something of the general nature of management by exception. For management by exception provides high internal homogeneity and therefore tends to increase the density of forcible maps in an organization's control net. This in turn promotes useful, stable, organizational behavior in a wide variety of structural circumstances. Management by exception and management by sufficient indication are distinct styles, but in their extreme forms they are virtually identical. For its part, reducing k, the number of sources of control in an organization's control net, increases the density of forcible maps and the tractability of behavior, but at the cost of variety

in structural options. Where sources of control are high, tractable system behavior can be obtained, on the average, without modification of the net structure provided relatively high intensities of management by exception, or of management by sufficient indication predominate.

In concluding we note that neither internal homogeneity nor forcibility model management by consensus or aggregation of information. Interpreting these styles requires the notion of a threshold for mappings in a control system. We consider this concept and its ramifications in a subsequent paper.

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By describing the organizational analogues corresponding to a binary switching net, we suggest that switching net models provide useful insight into the behavior of complex organizational control systems. Imposition of certain types of structure on the responses of elements in a system to their inputs enables control of the overall behavior of such nets to an extent that makes them plausible as real world models. We examine two such structural concepts, internal homogeneity and forcibility, with respect to their influence on the behavior of individual elements and of the system as a whole. We then interpret these two concepts as managerial strategies, the former being management by exception and the latter being management by priority.

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